Time limit: 60 minutes
Instructions: For this test, you work in your teams of three to solve 15 short answer questions.
No calculators.

Short Answer Questions: For the short answer questions, all answers are integers between -2147483648 and 2147483647, inclusive. Your team should submit only one answer form. Only the first answer form received from your team will be considered for grading.

1. There are 12 seats in a circle, with one person in each seat. Simultaneously, every person moves to one of the two seats adjacent to them, with equal probability. If the probability that no two people move to the same seat can be written as $\frac{p}{q}$ for relatively prime integers p and q, find $p+q$.
2. Find the sum of all positive integers n such that $\lfloor \sqrt[n]{1010} \rfloor < \lfloor \sqrt[n]{2020} \rfloor$.
3. Find the number of integers k for which k^{666} has less than 1000 digits.
4. If $x+y=6$ and $x^3+y^3=108$, find x^5+y^5.
5. If x, y, z are real numbers selected uniformly at random in the interval $[0,2]$, and the probability that $x+y+z \leq \sqrt[3]{5}$ can be written as $\frac{p}{q}$ for relatively prime integers p and q, find $p+q$.
6. Let ABC be a triangle with $AB=6, AC=8, BC=10$. Construct rectangles $BCDE$, $ABFG$, and $CAHI$ outside triangle ABC. If $BE=4$ and points D, E, F, G, H, I lie on a circle, and the area of the circle is $k\pi$, find k.
7. The maximal odd divisor of an integer n is $\frac{n}{2^k}$, where k is the largest possible value such that $\frac{n}{2^k}$ is an integer. Find the sum of the maximal odd divisors for positive integers between 1 and 100, inclusive.
8. Gauss's teacher was unable to stump Gauss with the sum of the first 100 numbers, so he turned to the multiplicative version of it: factorials. He defines $F_n(x)$ to be the largest integer p such that n^p divides x, and asks Gauss to find $F_{10}(100^{100} \cdot 99^{99} \cdot 98^{98} \cdots 3^3 \cdot 2^2 \cdot 1^1) - F_{10}(100! \cdot 99! \cdot 98! \cdots 3! \cdot 2! \cdot 1!)$. What does he get?
9. Ashwin picks a point $P=(a,b)$ in the interior of the unit circle uniformly at random and draws the line l_1 connecting P and $(0,1)$, and the line l_2 connecting P and $(0,-1)$. The probability that either the slope of line l_1 or the slope of l_2 is less than 3 in absolute value can be written as $1-\frac{p}{q\pi}$, where p and q are relatively prime positive integers. Find p^2+q^2.
10. The decimal notation of $\frac{1}{17}$ is the repeating decimal

$$0.\overline{058823529A1176BC},$$

which has 16 digits in the repetend, or the part which repeats. Find the three-digit integer $\overline{ABC}$.

11. Sarah is bored one day and decides to walk around the unit circle. She starts at $P(1,0)$ and walks a distance of π in the counterclockwise direction. She then turns around, and walks a distance of $\frac{2\pi}{3}$. She turns around again, and walks a distance of $\frac{2\pi}{9}$. She turns around again, and walks a distance of $\frac{4\pi}{27}$. After the nth turn, she walks a distance of $\frac{2^{\frac{n+1}{2}}}{3^n}\pi$ if n is odd, and a distance of $(\frac{2}{9})^{n/2}\pi$ if n is even. After walking for a long time, Sarah becomes arbitrarily close to a point Q, where $m\angle POQ = \theta$. If $\angle POQ = \frac{p}{q}\pi$ for relatively prime integers p and q, find $p+q$.

12. Find the integer k between 0 and 99 inclusive such that $k \equiv \frac{2^{2020}-1}{15} \pmod{100}$.

13. Let $f(n)$ be the number of ordered triples of nonnegative integers (a,b,c) with $2a+b+c=n$. If

$$\sum_{n=0}^{\infty} \frac{f(n)}{2^n}$$

can be written as $\frac{p}{q}$ for relatively prime integers p and q, find $p+q$.

14. Let ABC be an acute triangle with orthocenter H and circumcenter O. Let M be the midpoint of arc BC not containing A. Suppose that $AB = 10, AC = 12$, and $HA = HM$. Find the square of the length of BC.

15. Find the number of functions $f : \{1,2,\ldots,15\} \to \{1,2,\ldots,15\}$ satisfying the property that $\gcd(f(x), f(y)) = \gcd(x,y)$ for all $x \neq y$.