

Time limit: 10 minutes

Instructions: For this test, you will individually solve two short answer questions.

No calculators.

1. Find the positive integer x such that

$$\log_5 12, \log_5 x, \log_5 108$$

is an arithmetic sequence in that order.

2. A rhombus has side length 10 and area 80. A circle is inscribed in the rhombus. If the area of the circle can be written as $k\pi$, find k .

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3. If $f(x)$ is a linear function and $f(f(f(x))) = 8x + 693$, find $f(1)$.

4. Find the number of permutations of $1, 2, 3, \dots, 10$ such that for all integers x with $1 \leq x \leq 5$, in the permutation x appears to the left of $2x$.

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5. Moor takes a triangular sheet of paper and lays it over another (not necessary congruent) triangular sheet of paper. The resulting polygon formed by the two triangles together has n sides. What is the sum of all possible values of n ?

6. How many values x satisfy $0 \leq x < 1001$ and $x^3 - 1000x^2 - 9x + 992 \equiv 0 \pmod{1001}$?

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7. Maira is interested in constructing quadratics out of each other and starts with a relatively simple example. She takes two quadratics: $(x + m)^2$ and $(x + m + 2)^2$, and takes their average (adding both together and dividing the sum by 2). She takes her new quadratic and finds the roots r_m and s_m . Finally, she calculates the remainder of $\sum_{m=1}^{2020} (r_m^2 + s_m^2)$ when it is divided by 1000. What value does she get?

8. For positive integers n and b , let $f_b(n)$ denote the sum of the digits of the base b representation of n . How many positive integers $n \leq 2020$ satisfy $f_2(n) = f_4(n)$?

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9. Two elements are selected at random from the set $\{1, 2, 3, \dots, 2020\}$. If the expected value of the larger of the two elements is $\frac{m}{n}$ in lowest terms, find $m + n$.

10. Sam writes all positive divisors of 5040 (including itself) on a whiteboard. Then, in one turn, he picks one of the numbers on the whiteboard uniformly at random, and then erases that number and all multiples of it. He keeps doing this until all numbers are erased. If the expected number of turns taken by this process is $\frac{m}{n}$ in lowest terms, find $m + n$.